Throughput Analysis of Slotted Aloha with Retransmission Limit in Fading Channels

Peng Li[1], Yitong Li[1(✉)], Wen Zhan[2], Pei Liu[3,4,5], Yue Zhang[6], Dejin Kong[7], and Kehao Wang[3,5]

[1] School of Information Engineering, Zhengzhou University, Zhengzhou 450001, China
ieytli@zzu.edu.cn

[2] School of Electronics and Communication Engineering, Sun Yat-sen University, Shenzhen 510006, China
zhanw6@mail.sysu.edu.cn

[3] School of Information Engineering, Wuhan University of Technology, Wuhan 430070, China
pei.liu@ieee.org, kehao.wang@whut.edu.cn

[4] Key Lab of Broadband Wireless Communication and Sensor Network Technology (Nanjing University of Posts and Telecommunications), Ministry of Education, Nanjing 210003, China

[5] Hubei Key Laboratory of Broadband Wireless Communication and Sensor Networks, Wuhan University of Technology, Wuhan 430070, China

[6] College of Engineering, Shantou University, Shantou 515063, China
yzhang742-c@my.cityu.edu.hk

[7] School of Electronic and Electrical Engineering, Wuhan Textile University, Wuhan 430200, China
djkou@wtu.edu.cn

Abstract. In practical random access networks, a packet discard scheme is usually applied to alleviate the network congestion, where a packet will be discarded if its retransmission times reaches the limit K. In this paper, the effect of retransmission limit K on the throughput performance of slotted Aloha over Rayleigh fading channels is studied. Explicit expression of the maximum throughput is obtained as a function of the SINR threshold μ and mean received SNR ρ. The analysis shows that though the maximum throughput is independent of the retransmission limit K, the corresponding optimal initial transmission probability of nodes has to cautiously selected in accordance with the retransmission limit K and the backoff function $\{\omega_i\}_{i=0,\ldots,K-1}$.

1 Introduction

In recent years, the rising of Internet of Things (IoT) paradigm facilitates the Machine-Type Communications (MTC), which enables automated information

Q. Liang et al. (Eds.): CSPS 2021, LNEE 878, pp. 766–774, 2022.
https://doi.org/10.1007/978-981-19-0390-8_95

exchange among massive number of machine-type devices. Due to the short and bursty traffic nature of MTC, random access protocols, such as slotted Aloha and CSMA, have been regarded as the most competitive solution in Medium Access Control (MAC) layer. Specially, slotted Aloha provide a simple way for multiple nodes to share the channel, in which each node can transmit at the beginning of the time slot when it has packet to send, and retransmitted according to the adopted backoff policy if it involves in a collision. Due to its simplicity, Aloha-type random access methods have been widely applied in practical machine-machine (M2M) networks, such as the Low power Wide Area Networks (LPWAN) including NB-IoT, Weightless and LoRaWAN [1–3].

The early studies on slotted Aloha focused on the collision model [4], in which a packet can be correctly received only when there is no concurrent transmissions and the channel condition was assumed to be perfect. While with channel fading effect is taken into consideration, the capture model is more appropriate. Under the capture model, each node's packet can be correctly recovered if its received SINR is above a certain threshold [5,6]. Therefore, capture model has the capability to correctly receive multiple packets if the SINR threshold is sufficiently low [5–7].

It has been long observed that random access may result in severe congestion when the network traffic load is heavy. With large offered load, huge amount of nodes will contend for the shared channel, which results in frequent collisions. The retransmission of the collided packets will further aggravate the channel contention level. Therefore, a packet discard mechanism is usually applied to alleviate the congestion, where a packet is discarded if its retransmission times reaches the retransmission limit K. With the collision model, the stability issue of slotted Aloha with retransmission limit was studied in [8,10]. [8] explored the effect of retransmission limit on the stability region for two asymmetric nodes. The effect of retransmission limit on the CSMA with collision model was studied in [10], in which the explicit expressions of both throughput and mean access delay are obtained as the functions of the retransmission limit.

In this paper, the throughput performance of a buffered slotted Aloha with retransmission limit K in Rayleigh fading channels is studied under the capture model. First of all, both the steady-state point and throughput in unsaturated and saturated conditions are derived by analyzing the head-of-line (HOL) packets' aggregate activities. Explicit expression of the maximum throughput is also derived as a function of μ and ρ, which is independent on the retransmission limit K. While the optimal initial transmission probability to achieve the maximum throughput is closely related to the retransmission limit K and the backoff function $\{\omega_i\}_{i=0,\ldots,K-1}$.

The paper is organized as follows. In Sect. 2, the system model is described. Section 3 demonstrates how to obtain the steady-state points in both unsaturated and saturated conditions. The throughput analysis is provided in Sect. 4. Conclusion and future work are presented in Sect. 5.

2 System Model

In this paper, a slotted Aloha network with packet retransmission limit over Rayleigh fading channels is considered. Specifically, we assume the multiple access scenario where n nodes communicate with a single access point. Each node can only start the transmission attempt at beginning of a time slot and the transmission time of each packet is assumed to be one time slot. Each node has the same packet input rate of $\lambda \in (0, 1]$ and an infinite buffer size. For the head-of-line (HOL) packet of each node's queue, it will be discarded if its transmission fails for $K \in [1, \infty)$ consecutive times, where K denotes the retransmission limit. Moreover, we assume that the uplink power control is performed to mitigate the near-far effect. As a result, each node has the same mean received SNR ρ. With the capture model, a packet can be successfully received if its received SINR is no smaller than the threshold μ.

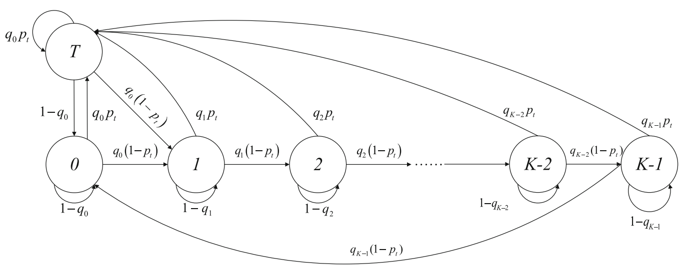

Fig. 1. Diagram illustration of the HOL-packet's state transition process.

2.1 HOL-packet Model

The performance of Aloha networks is closely related to the aggregate activities of HOL packets. As illustrated in Fig. 1, the HOL packet's behavior can be modeled as a discrete-time Markov process. Specifically, a newly generated HOL packet is in State T with probability q_0 to transmit. If it is not transmitted, it will transfer to State 0 with probability $1 - q_0$. A phase-i HOL packet has the transmission probability with q_i for $i = 1, 2, ..., K-1$, where the phase defines the number of unsuccessful transmission of the HOL packet. Therefore, for a phase-i HOL packet, $i = 1, 2, ..., K-2$, it would stay in State i if it is not transmitted with probability $1-q_i$. Otherwise, it moves to State $\min(K-1, i+1)$ if its transmission attempt is unsuccessful, or State T if the transmission is successful. Specially, a HOL packet with phase $K-1$ will transfer to State T if its retransmission, i.e., the $(K-1)$−th retransmission, is successful. If its transmission fails, it will be discarded and moves to State 0, indicating a new packet becomes the HOL packet and is ready for transmission. Note that in random access networks, when the nodes involve in frequent collisions, their transmission probabilities should be reduced to alleviate the channel congestion. For demonstration, assume that $q_i = q_0 \cdot \omega_i$ where $q_0 \leq 1$ is referred as the initial transmission probability and ω_i is referred as the backoff function with $\omega_0 = 1$ and $\omega_i \geq \omega_{i+1}$, $i = 0, 1, \ldots, K-1$. Therefore, $\{q_i\}_{i=0,1,\ldots,K-1}$ are monotonic non-increasing with i.

The steady-state distribution $\{\pi_i\}$ of the Markov chain is given by

$$\pi_T = \frac{1-(1-p)^K}{\sum\limits_{i=0}^{K-1} \frac{(1-p)^i}{q_i}}, \tag{1}$$

and

$$\begin{cases} \pi_0 = \frac{1-q_0(1-(1-p)^K)}{q_0(1-(1-p)^K)} \pi_T, \\ \pi_i = \frac{(1-p)^i}{q_i(1-(1-p)^K)} \pi_T, \quad \text{for } i = 1, 2, \ldots K-1, \end{cases} \tag{2}$$

where $\lim\limits_{t \to \infty} p_t = p$. Note that the HOL packet is discarded if its transmission is unsuccessful for K times. Therefore, p_d can be expressed as

$$p_d = (1-p)^K. \tag{3}$$

As each HOL packet has probability $1 - p_d$ to be served with service rate of π_T. The offered load of each node's queue can be written as

$$\delta = \frac{\lambda(1-p_d)}{\pi_T}. \tag{4}$$

3 Steady-State Points

As illustrated in Sect. 2.1, the network performance is determined by the steady-state probability of successful transmission of HOL packets p. By following the same derivation in [7], p can be expressed as

$$p = \sum_{i=0}^{n-1} r_i \cdot \Pr\{i \text{ concurrent transmissions}\}. \tag{5}$$

Specifically, r_i denotes the conditional probability that the tagged HOL packet's transmission is successful given that there exists i concurrent transmissions, which can be explicitly written as [6]

$$r_i = \frac{\exp\left(-\frac{\mu}{\rho}\right)}{(\mu+1)^i}, \tag{6}$$

The probability that there exists i concurrent transmissions among the remaining $n-1$ nodes is closely determined by the offered load δ, based on which the network condition can be divided into unsaturated condition and saturated condition. In the unsaturated condition, the offered load δ is lower than 1 and each node's queue has the probability $1 - \delta$ to be empty. On the other hand, in the saturated condition, each node is always busy with a HOL packet to serve. In this section, the steady-state points in these two conditions will be characterized.

3.1 Steady-State Point in Unsaturated Condition p_L

In the unsaturated condition, for each node, the probability that it has a HOL packet requesting transmission is written as $\delta(\pi_T q_0 + \sum_{i=0}^{K-1} \pi_i q_i) = \frac{\lambda(1-p_d)}{p}$. Therefore, the probability that there exists i concurrent transmissions is given by

$$\Pr\{i\,\text{concurrent transmissions}\} = \binom{n-1}{i}\left(1-\frac{\lambda(1-p_d)}{p}\right)^{n-1-i} \cdot \left(\frac{\lambda(1-p_d)}{p}\right)^i. \qquad (7)$$

By combining (5), (6) and (7), we can obtain p in the unsaturated condition as

$$\begin{aligned} p &= \sum_{i=0}^{n-1} \frac{\exp\left(-\frac{\mu}{\rho}\right)}{(\mu+1)^i} \cdot \binom{n-1}{i} \left(1-\frac{\lambda(1-p_d)}{p}\right)^{n-1-i} \left(\frac{\lambda(1-p_d)}{p}\right)^i \\ &= \exp\left(-\frac{\mu}{\rho}\right) \cdot \left(1-\frac{\mu}{\mu+1} \cdot \frac{\lambda(1-p_d)}{p}\right)^{n-1} \overset{\text{for large } n}{\approx} \exp\left\{-\frac{\mu}{\rho} - \frac{\mu}{\mu+1} \cdot \frac{\hat{\lambda}(1-p_d)}{p}\right\}, \end{aligned} \qquad (8)$$

where $\hat{\lambda} = n\lambda$ denotes the aggregate input rate. The explicit expressions of the roots in (8) is hard to obtain. Following a similar approach in [10], it can be easily shown that (8) has either one single non-zero root $0 < p_L \leq 1$ or two non-zero roots $0 < p_S \leq p_L \leq 1$, which is closely determined by the SINR threshold μ, mean received SNR ρ and retransmission limit K. Only p_L is the steady-state point in unsaturated condition.

3.2 Steady-State Point in Saturated Condition p_A

As the node input rate increases, the network will eventually become saturated. In the saturated condition, each node always has a HOL packet to transmit. The probability that a HOL packet is transmitted can be written as $\pi_T q_0 + \sum_{i=0}^{K-1} \pi_i q_i = \frac{\pi_T}{p}$. By combining (5) and (6), we can obtain p in the saturated condition as

$$p = \sum_{i=0}^{n-1} \frac{\exp\left(-\frac{\mu}{\rho}\right)}{(\mu+1)^i} \cdot \binom{n-1}{i} \left(1-\frac{\pi_T}{p}\right)^{n-1-i} \left(\frac{\pi_T}{p}\right)^i \overset{\text{for large } n}{\approx} \exp\left\{-\frac{\mu}{\rho} - \frac{n\mu}{\mu+1} \cdot \frac{1-(1-p)^K}{\sum_{i=0}^{K-1} \frac{p(1-p)^i}{q_i}}\right\}. \qquad (9)$$

With the similar approach proposed in [7,10], it can be easily shown that (9) has one single root $0 < p_A < 1$ when the transmission probabilities $\{q_i\}_{i=0,\ldots,K-1}$ are monotonic non-increasing with i. Therefore, we can see that the slotted Aloha network has two steady-state points, p_L and p_A, which are referred as desired steady-state point and undesired steady-state point, respectively.

3.3 Absolute-Stable Region

From (8) and (9), it can be observed the desired steady-state point p_L is determined by the aggregate input rate $\hat{\lambda}$ while insensitive to the transmission probabilities $\{q_i\}_{i=0,\ldots,K-1}$. On the other hand, the undesired steady-state point p_A is closely related to the transmission probabilities $\{q_i\}_{i=0,\ldots,K-1}$. In this subsection, an absolute-stable region $S_L = [q_l, q_u]$ is derived. With the initial transmission probability $q_0 \in S_L$, the network is guaranteed to operate at the desired steady-state point p_L. On the other hand, if $q_0 \notin S_L$, the network may shift to the undesired steady-state point p_A. By following the similar approach proposed in [9], we can obtain that if (8) has one non-zero root p_L, the absolute-stable region S_L is given by

$$S_L^{(1)} = \left[\lambda \sum_{i=0}^{K-1} \frac{(1-p_L)^i}{\omega_i}, 1\right]. \tag{10}$$

While if (8) has two non-zero roots, p_S and p_L, the absolute-stable region S_L reduces to

$$S_L^{(2)} = \left[\lambda \sum_{i=0}^{K-1} \frac{(1-p_L)^i}{\omega_i}, -\frac{\mu+1}{n\rho} - \frac{\mu+1}{n\mu} \ln p_S\right], \tag{11}$$

which is closely related to the node input rate λ and retransmission limit K.

Figure 2 illustrates how the steady-state points vary with the initial transmission probability $q_0 \in (0,1]$ under retransmission limit $K = 1$ and $K = 20$ when μ and ρ are fixed. When $K = 1$, (8) has only one non-zero root p_L and the absolute-stable region is given by $S_L^{K=1} = [0.012, 1]$. Therefore, the network operates at $p = p_L$ for $q_0 \in S_L^{K=1}$, and it is insensitive to the initial transmission probability q_0. As K increases, for instance, when $K = 20$, the absolute-stable region reduces to $S_L^{K=20} = [0.024, 0.063]$. As the transmission probability q_0 increases, when $q_0 \notin S_L^{K=20}$, the steady-state point will shift to the undesired steady-state point p_A, which sharply diminishes as q_0 increases, as illustrated in Fig. 2.

4 Throughput Analysis

When the network is unsaturated, it is operating at the undesired steady-state point p_L. The corresponding throughput is given by

$$\hat{\lambda}_{out}^u = (1 - p_d^{p_L})\hat{\lambda}. \tag{12}$$

Specifically, $p_d^{p_L} = (1-p_L)^K$ is not only the packet discard ratio, but also denotes the throughput loss ratio due to the packet discard. By combining (8) and (12), $\hat{\lambda}_{out}^u$ can be further written as $\hat{\lambda}_{out}^u = \frac{\mu+1}{\mu}\left(-p_L \ln p_L - p_L \frac{\mu}{\rho}\right)$. By taking the fist-order derivative with respect to p_L, we can then obtain that the maximum throughput $\hat{\lambda}_{\max}^u$ in the unsaturated condition as

$$\hat{\lambda}_{\max}^u = \frac{\mu+1}{\mu} \exp\left(-1 - \frac{\mu}{\rho}\right), \tag{13}$$

which is achieved when $p_L = \exp\left(-1 - \frac{\mu}{\rho}\right)$. As the input rate of each node λ grows and exceeds the service rate π_T, each node's queue is always busy and the network becomes saturated. In this case, the throughput is equal to the aggregate service rate $n\pi_T^{p=p_A} = \frac{n(1-(1-p)^K)}{\sum_{i=0}^{K-1}(1-p)^i/q_i}$, which varies with the transmission probabilities q_i. According to (9), the throughput $\hat{\lambda}_{out}^s$ in the saturated condition is further written as $\hat{\lambda}_{out}^s = \frac{\mu+1}{\mu}\left(-p_A \ln p_A - p_A \frac{\mu}{\rho}\right)$, which is maximized with

$$\hat{\lambda}_{\max}^s = \frac{\mu+1}{\mu}\exp\left(-1-\frac{\mu}{\rho}\right), \tag{14}$$

when $p_A = \exp\left(-1-\frac{\mu}{\rho}\right)$ and the initial transmission probability q_0 is set as

$$q_0^* = \frac{\mu+1}{n\mu} \cdot \frac{\sum_{i=0}^{K-1} \exp\left(-1-\frac{\mu}{\rho}\right)\left(1-\exp\left(-1-\frac{\mu}{\rho}\right)\right)^i/\omega_i}{1-\left(1-\exp\left(-1-\frac{\mu}{\rho}\right)\right)^K}. \tag{15}$$

According to (13) and (14), we can conclude that the maximum throughput is given by $\hat{\lambda}_{\max} = \hat{\lambda}_{\max}^s = \hat{\lambda}_{\max}^u = \frac{\mu+1}{\mu}\exp\left(-1-\frac{\mu}{\rho}\right)$, which is solely determined by the SINR threshold μ and the mean received ρ. While the optimal transmission probability q_0^* achieving $\hat{\lambda}_{\max}$ is also determined by the retransmission limit K and the backoff function $\{\omega_i\}_{i=0,1,\ldots,K-1}$.

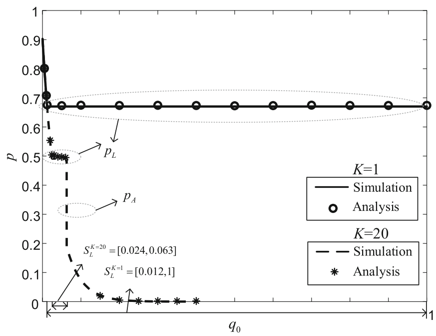

Fig. 2. Steady-state points p versus initial transmission probability q_0 for retransmission limit $K = 1$ and $K = 20$. $n = 50$, $\hat{\lambda} = 0.6$, $\mu = 1$, $\rho = 10$ dB and $\omega_i = 1$ for $i = 0, 1, \ldots K-1$.

Figure 3 illustrates the throughput performance versus the initial transmission probability $q_0 \in (0, 1]$ with retransmission limit $K = 1$ and $K = 20$. With $K = 1$, it can be observed that with a low aggregate input rate, the network operates at the desired steady-state point p_L if $q_0 \in S_L^{K=1} = [0.012, 1]$, and the corresponding throughput $\hat{\lambda}_{out}^u$ is given by (12), which does not vary with $q_0 \in S_L^{K=1}$. As retransmission limit K increases, for instance, from $K = 1$ to $K = 20$, as the throughput loss ratio due to packet dropping is reduced, the throughput $\hat{\lambda}_{out}^u$ in the unsaturated condition increases.

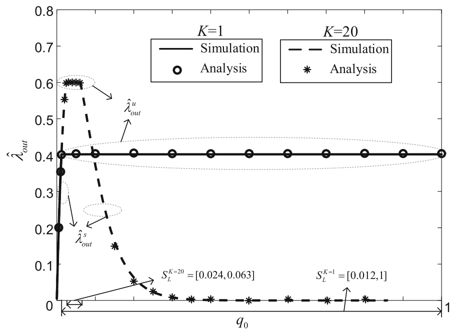

Fig. 3. Throughput $\hat{\lambda}_{out}$ versus initial transmission probability q_0. $n = 50$, $\hat{\lambda} = 0.6$, $\mu = 1$, $\rho = 10$dB and $\omega_i = 1$ for $i = 0, 1, ...K - 1$.

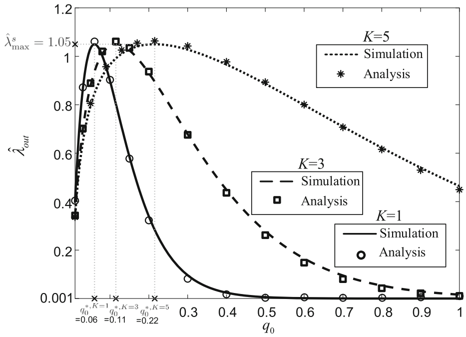

Fig. 4. Throughput $\hat{\lambda}_{out}$ versus initial transmission probability q_0. $n = 50$, $\hat{\lambda} = 2$, $\mu = 0.5$, $\rho = 10$ dB and $\omega_i = \frac{1}{2^i}$ for $i = 0, 1, ...K - 1$.

Figure 4 illustrates the throughput performance versus the initial transmission probability q_0 for retransmission limit $K = 1$ and $K = 20$ when the input rate of each node is pushed to the limit. Here, the network is fully saturated for $q_0 \in (0, 1]$. It can be clearly seen that the maximum throughput can be achieved for different retransmission limit K remain the same. Yet the corresponding optimal transmission probability q_0^* varies according to retransmission limit K.

5 Conclusion

This paper presents the throughput analysis of slotted Aloha networks with retransmission limit K under the capture model. The maximum throughput $\hat{\lambda}_{\max}$ is derived as an explicit expression of SINR threshold μ and mean received SNR ρ. The optimal setting of initial transmission probability q_0 to achieve $\hat{\lambda}_{\max}$ is also obtained, which is closely determined by the retransmission limit K and the backoff function $\{\omega_i\}_{i=0,1,...,K-1}$. Note that the effect of retransmission limit on the delay performance of random access is also significant. Intuitively, with a small retransmission limit, a collided HOL packet would has a large probability to be dropped. Therefore, the time for retransmission would be shortened, and thus both the access delay and queueing delay performance can be improved. In the future work, we will focus on the effect of retransmission limit on the delay performance of random access networks.

Acknowledgments. This work was supported in part by National Natural Science Foundation of China under Grant 61801433, Grant 62001524, Grant 62001336, Grant 62001333, and Grant 61911540481, in part by the Science, Technology and Innovation Commission of Shenzhen Municipality under Grant 2021A04, in part by the open research fund of Key Lab of Broadband Wireless Communication and Sensor Network Technology (Nanjing University of Posts and Telecommunications), Ministry of Education under Grant JZNY202105, in part by the Fundamental Research Funds for the Central Universities, Sun Yat-sen University, No. 2021qntd11, and in part by the Fundamental Research Funds for the Central Universities under Grant WUT: 2021IVB018.

References

1. Laya, A., Alonso, L., Zarate, J.: Is the random access channel of LTE and LTE-A suitable for M2M communications? A survey of alternatives. IEEE Commun. Surveys Tuts. **16**(1), 4–16 (2014)
2. Laya, A., Kalalas, C., Vazquez-Gallego, F., Alonso, L., Alonso-Zarate, J.: Goodbye, ALOHA! IEEE Access **4**, 2029–2044 (2016)
3. Yang, W., et al.: Narrowband wireless access for low-power massive internet of things: a bandwidth perspective. IEEE Wireless Commun. **24**(3), 138–145 (2017)
4. Abramson, N.: Packet switching with satellites. In: Proceedings of the National Joint Computing Conference, vol. 42, pp. 695–702, Montvale 1973
5. Luo, J., Ephremides, A.: Power levels and packet lengths in random multiple access with multiple-packet reception capability. IEEE Trans. Inf. Theory **52**, 414–420 (2006)
6. Dua, A.: Random access with multi-packet reception. IEEE Trans. Wireless Commun. **7**(6), 2280–2288 (2008)
7. Li, Y., Dai, L.: Maximum sum rate of slotted Aloha with capture. IEEE Trans. Commun. **64**(2), 690–705 (2016)
8. Seo, J.B., Hu, J., Leung, V.C.M.: Impacts of retransmission limit on stability and throughput regions of S-ALOHA systems. IEEE Trans. Veh. Technol. **11**(66), 10296–10306 (2017)
9. Dai, L.: Stability and delay analysis of buffered Aloha networks. IEEE Trans. Wireless Commun. **11**(8), 2707–2719 (2012)
10. Sun, X., Dai, L.: Performance optimization of CSMA networks with a finite retry limit. IEEE Trans. Wireless Commun. **15**(9), 5947–5962 (2016)